CLASSICAL MOTION OF A FREE SPIN

TRANSLATION $\vec{F} = \frac{d\vec{p}}{dt}$

ROTATION. $\vec{\tau} = \frac{d\vec{L}}{dt} = (\vec{\mu} \times \vec{H})$

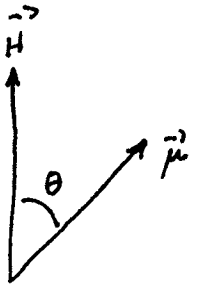
$\vec{\mu} = \gamma \vec{L}$

$$\frac{d\vec{\mu}}{dt} = \gamma \vec{\mu} \times \vec{H}$$

$$d\vec{\mu} = \gamma (\vec{\mu} \times \vec{H})\, dt$$

$d\vec{\mu} \perp \vec{\mu}$

$d\vec{\mu} \perp \vec{H}$



$\Rightarrow$ cone of rotation with fixed θ.

For $\vec{H} = H_0 \hat{z}$ $\quad H_x = H_y = 0 \quad H_z = H_0$

$$\mu_x(t) = A \cos(\omega_0 t + \phi)$$

$$\mu_y(t) = A \sin(\omega_0 t + \phi)$$

$$\mu_z(t) = B$$

$$\omega_0 = \gamma H_0$$

$$A^2 + B^2 = \mu^2$$

Classically,

$$\tan\theta = \frac{A}{B}$$

θ can have any value

Quantum mechanically

$$\cos\theta = \frac{m_s}{\sqrt{s(s+1)}}$$

for many spins $m_s \to \infty$, recover classical result.

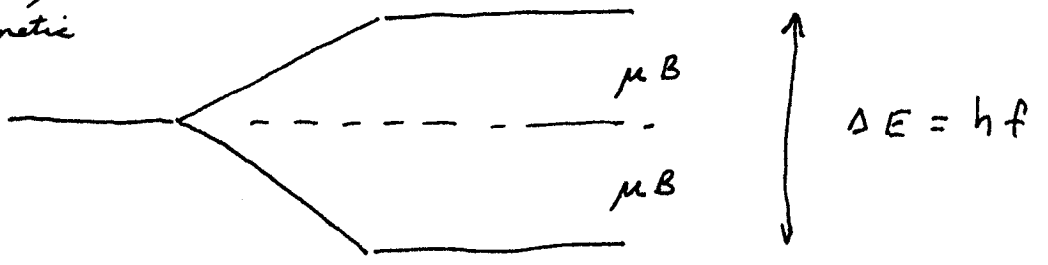
QUANTUM SOLUTION FOR FREE SPINS

$$H|\psi\rangle = -i\hbar \frac{d}{dt}|\psi\rangle$$

$$H = -\vec{\mu}\cdot\vec{B} = -\gamma \vec{S}\cdot\vec{B}$$

$\vec{\mu} = \gamma \vec{S}$

γ: gyromagnetic ratio



$$hf = 2\mu B$$

$$f = \left(\frac{2\mu}{\hbar}\right)B$$

$\vec{B} = B_0 \hat{z}$

p: 4.26 4257.7

e: 2.80

$$H = -\gamma B_0 S_z = -\gamma B_0 \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\vec{m}$ ⇄ $e\vec{v}$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $-\gamma B_0 \hbar/2$ low energy when $\vec{S} \parallel \vec{B}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $+\gamma B_0 \hbar/2$

For $|\psi(0)\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$

$$|\psi(0)\rangle = a\,|\uparrow\rangle + b\,|\downarrow\rangle$$

$$|\psi(t)\rangle = a\,e^{-iE_1 t/\hbar}\,|\uparrow\rangle + b\,e^{-iE_2 t/\hbar}\,|\downarrow\rangle$$

$$= a\,e^{i\gamma B_0 t/\hbar}\,|\uparrow\rangle + b\,e^{-i\gamma B_0 t/\hbar}\,|\downarrow\rangle$$

normalized $\quad |a|^2 + |b|^2 = 1 \quad$ (real)

$$a = \cos(\alpha/2)$$

$$b = \sin(\alpha/2)$$

$$|\psi(t)\rangle = \begin{pmatrix} \cos(\alpha/2)\,e^{i\gamma B_0 t/2} \\ \sin(\alpha/2)\,e^{-i\gamma B_0 t/2} \end{pmatrix}$$

$$\langle S_x \rangle = \langle \psi(t) | S_x | \psi(t) \rangle$$

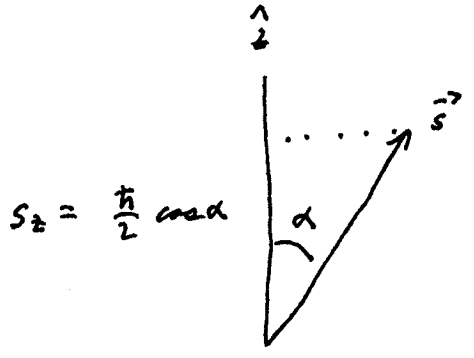
$$(a^* \; b^*) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$= \frac{\hbar}{2} \sin(\alpha)\cos(\gamma B_0 t)$$

$$\langle S_y \rangle = -\frac{\hbar}{2} \sin(\alpha) \sin(\gamma_0 B_0 t)$$

$$\langle S_z \rangle = \frac{\hbar}{2} \cos\alpha$$

$\omega = \gamma B_0$ LARMOR FREQUENCY



CLASSICALLY, α CAN HAVE ANY VALUE

QM, $\cos\alpha = \dfrac{m_s}{\sqrt{s(s+1)}}$

AS $m_s \to \infty$, RECOVER CLASSICAL MOTION

QM DRIVEN SPIN

$$H = -\vec{\mu} \cdot \vec{B}$$

$$= \frac{e}{mc}\left(H_0 S_z + H_1 \cos(\omega t) S_x + H_1 \sin(\omega t) S_y \right)$$

$$= \frac{e}{mc}\frac{\hbar}{2}\begin{pmatrix} H_0 & H_1 e^{-i\omega t} \\ H_1 e^{+i\omega t} & H_0 \end{pmatrix}$$

$$|\psi(t)\rangle = C_\uparrow(t)\,|\uparrow\rangle + C_\downarrow(t)\,|\downarrow\rangle$$

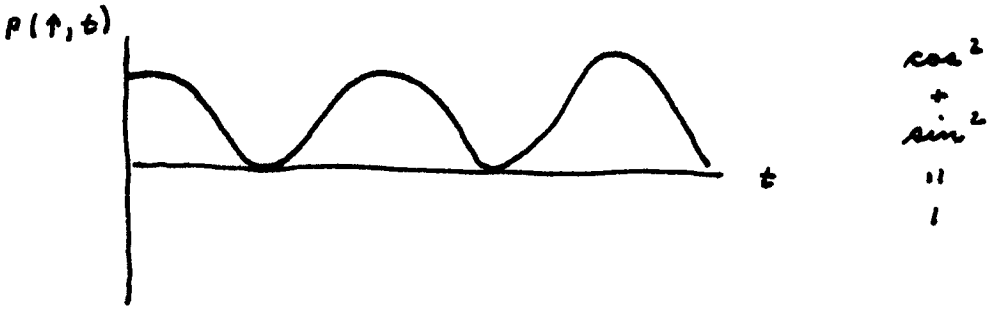
$$H|\psi\rangle = -i\hbar \frac{d}{dt}|\psi\rangle$$

$$\begin{pmatrix} H_0 & A \\ B & H_0 \end{pmatrix}\begin{pmatrix} C_\uparrow(t) \\ C_\downarrow(t) \end{pmatrix} = -i\hbar \begin{pmatrix} \dot{C}_\uparrow \\ \dot{C}_\downarrow \end{pmatrix}$$

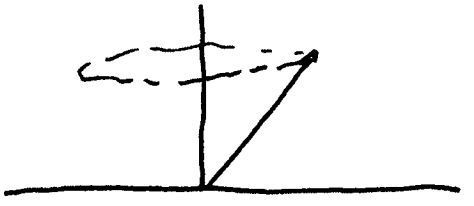
2 EQNS, 2 UNKNOWNS, + BC

$$\psi(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

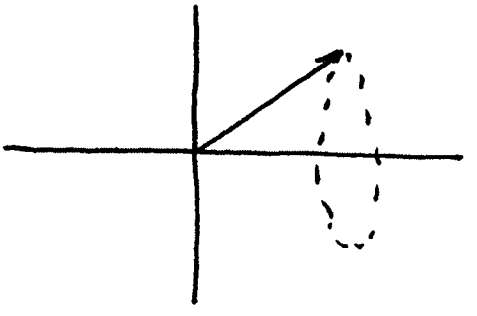
$$\psi(t) = \begin{pmatrix} \cos(\omega_1 t/2)\, \exp(-i\omega_0 t/2) \\ -\sin(\omega_1 t/2)\, \exp(+i\omega_0 t/2) \end{pmatrix}$$



$$\cos^2 + \sin^2 = 1$$



GO INTO ROTATING FRAME



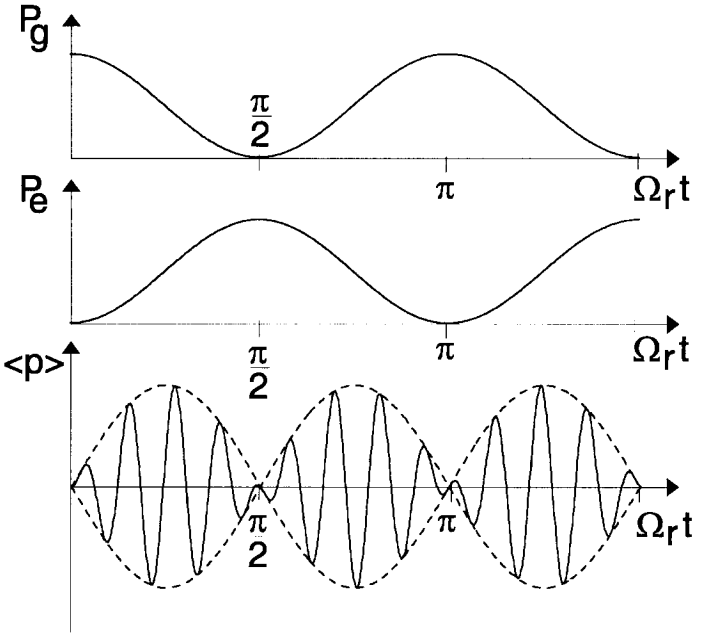


Figure 2.3: Evolution of occupation probabilities of ground and excited state and the average dipole moment of a two-level atom in resonant interaction with a coherent classical field.

The coherent external field drives the population of the atomic system between the two available states with a period $T_r = \pi/\Omega_r$. Applying the field only over half of this period leads to a complete inversion of the population. These Rabi-oscillations have been observed in various systems ranging from gases to semiconductors. Interestingly, the light emitted from the coherently driven two-level atom is not identical in frequency to the driving field. If we look at the Fourier spectrum of the polarization according to Eq.(2.95), we obtain lines at frequencies $\omega_\pm = \omega_{eg} \pm 2\Omega_r$. This is clearly a nonlinear output and the sidebands are called Mollow-sidebands [2] . Most important for the existence of these oscillations is the coherence of the atomic system over at least one Rabi-oscillation. If this coherence is destroyed fast enough, the Rabi-oscillations cannot happen and it is then impossible to generate inversion in a two-level system by interaction with light. This is the case for a large class of situations in light-matter interaction. So we are interested what happens in the case of loss of coherence due to additional interaction

$\vec{B}_0$ at the Larmor frequency.

• The $\pi/2$-*pulse,* for which $\vec{B}_1$ rotates at the Larmor frequency (*detuning* = 0) and for which $\vec{B}_1$ is switched on for a time duration $\tau_{\pi/2}$, such that $\Omega_R\ \tau_{\pi/2} = \pi/2$. As a result the magnetization is flipped by $\pi/2$ into the x-y plane.

• The π-*pulse,* for which $\vec{B}_1$ rotates at the Larmor frequency (*detuning* = 0) and for which $\vec{B}_1$ is switched on for a time duration τ_π, such that $\Omega_R\ \tau_\pi = \pi$. As a result the magnetization is flipped by π to the $-\hat{z}$ direction.

• *Magnetic resonance with relaxation*, for which the magnetization reaches a steady state. You can play with the value of the frequency detuning $\omega - \omega_0$. If $\omega \ll \omega_0$ or $\omega \gg \omega_0$, the effect of $\vec{B}_1$ is small and the steady state is close to the equilibrium magnetization along z. However, when $\omega \approx \omega_0$, the effect of $\vec{B}_1$ is important and the steady state is reached far from the z axis.

• A*diabatic following* occurs for $\omega \ll \omega_0$, in which case the magnetization precesses rapidly around the magnetic field and therefore follows the direction of $\vec{B}$ adiabatically.

Resize Images Rotate and Zoom in 3D

The time evolution of the ensemble magnetization $\vec{M} = (u, v, w)$ of an ensemble of magnetic moments in a magnetic field $\vec{B} = (B_x, B_y, B_z)$ is determined by the Bloch equations,

$$\frac{d}{dt}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \gamma \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} \times \begin{pmatrix} u \\ v \\ w \end{pmatrix} - \begin{pmatrix} \gamma_2 u \\ \gamma_2 v \\ \gamma_1 (w - w_{eq}) \end{pmatrix},$$

$$\frac{d\vec{M}}{dt} = +\gamma\, \vec{M} \times \vec{B} - \text{DAMPING}$$

where w_{eq} is the equilibrium z-component of $\vec{M}$, when all fields are 0; γ_1 and γ_2 are called the longitudinal and transverse relaxation rates, respectively. The total magnetic field is the vector sum of a static field $\vec{B}_0$ along z and a field $\vec{B}_1$ rotating in the x-y plane,

$$\vec{B} = \vec{B}_0 + \vec{B}_1 = \begin{pmatrix} B_1 \cos(\omega t) \\ B_1 \sin(\omega t) \\ B_0 \end{pmatrix}.$$

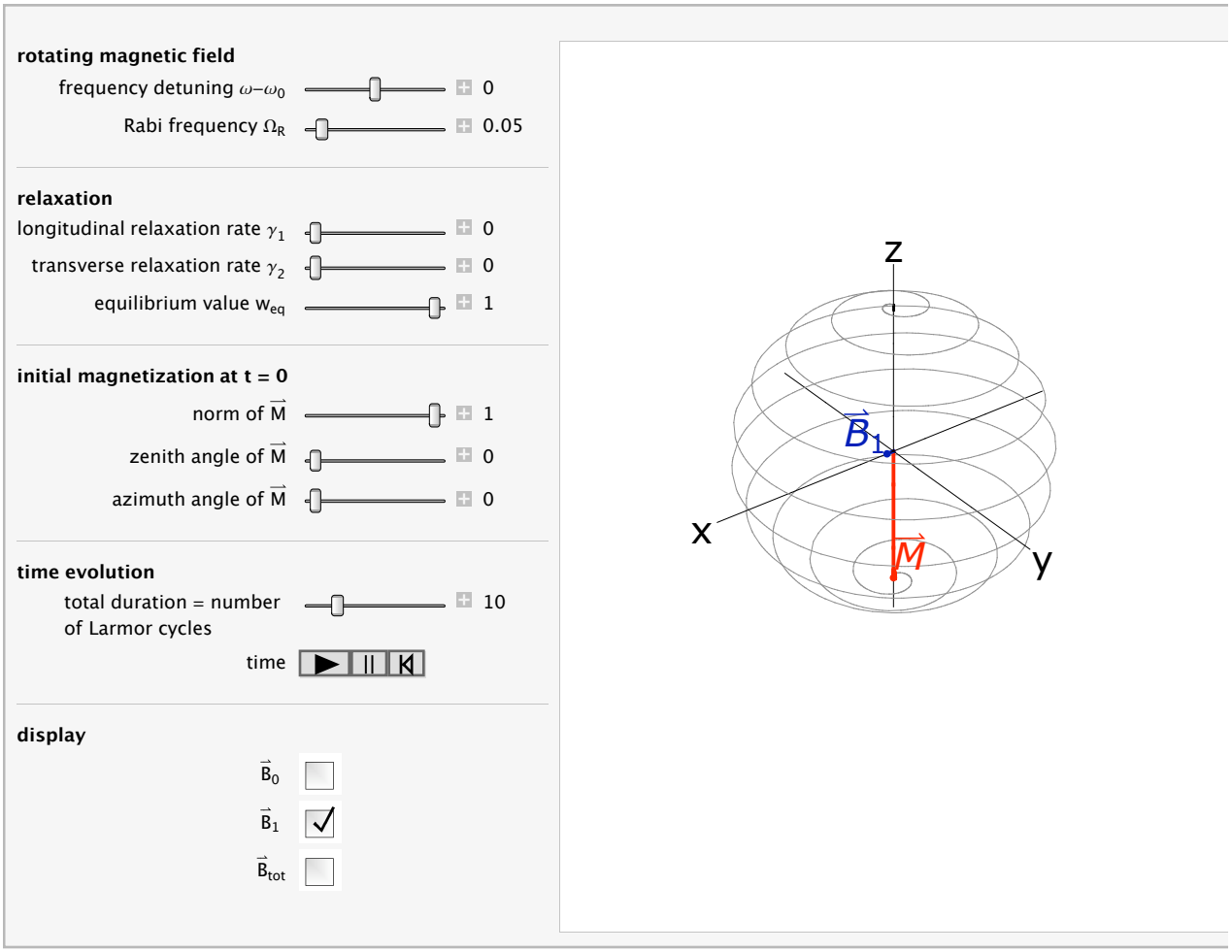
Inserting $\vec{B}$ into the Bloch equations yields

$$\frac{d}{dt}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} \Omega_R \cos(\omega t) \\ \Omega_R \cos(\omega t) \\ \omega_0 \end{pmatrix} \times \begin{pmatrix} u \\ v \\ w \end{pmatrix} - \begin{pmatrix} \gamma_2 u \\ \gamma_2 v \\ \gamma_1 (w - w_{eq}) \end{pmatrix},$$

$\omega_0 = \gamma\, |\vec{B}_0|$ LARMOR

$\Omega_R = \gamma\, |\vec{B}_1|$ RABI

Magnetic Resonance and Bloch Equations



This Demonstration visualizes the dynamics in the process of magnetic resonance in which the macroscopic magnetization $\vec{M}$ of an ensemble of paramagnetic particles is exposed to the common action of a static magnetic field $\vec{B}_0$ and a weak magnetic field $\vec{B}_1$ ($|\vec{B}_1| = \Omega_R/\gamma$) that rotates with a frequency ω around $\vec{B}_0$. The motion of $\vec{M}$ is governed by the so-called *Bloch equations*. The effect of $\vec{B}_1$ on $\vec{M}$ becomes most dramatic when the rotational frequency ω is equal to the Larmor free precession frequency $\omega_0 = \gamma\,|\vec{B}_0|$ (i.e., detuning $\omega - \omega_0 \approx 0$).

Many interesting solutions are registered as bookmarks, which you can activate by clicking the small cross at the upper right corner. For example:

• The *free Larmor precession,* which occurs for $\vec{B}_1 = 0$, so that magnetization precesses around

$\vec{B}_0$ at the Larmor frequency.

• The $\pi/2$-*pulse,* for which $\vec{B}_1$ rotates at the Larmor frequency (*detuning* = 0) and for which $\vec{B}_1$ is switched on for a time duration $\tau_{\pi/2}$, such that $\Omega_R\ \tau_{\pi/2} = \pi/2$. As a result the magnetization is flipped by $\pi/2$ into the x-y plane.

• The π-*pulse,* for which $\vec{B}_1$ rotates at the Larmor frequency (*detuning* = 0) and for which $\vec{B}_1$ is switched on for a time duration τ_π, such that $\Omega_R\ \tau_\pi = \pi$. As a result the magnetization is flipped by π to the $-\hat{z}$ direction.

• *Magnetic resonance with relaxation*, for which the magnetization reaches a steady state. You can play with the value of the frequency detuning $\omega - \omega_0$. If $\omega \ll \omega_0$ or $\omega \gg \omega_0$, the effect of $\vec{B}_1$ is small and the steady state is close to the equilibrium magnetization along z. However, when $\omega \approx \omega_0$, the effect of $\vec{B}_1$ is important and the steady state is reached far from the z axis.

• A*diabatic following* occurs for $\omega \ll \omega_0$, in which case the magnetization precesses rapidly around the magnetic field and therefore follows the direction of $\vec{B}$ adiabatically.

THINGS TO TRY

Resize Images ▪ Rotate and Zoom in 3D

DETAILS

The time evolution of the ensemble magnetization $\vec{M} = (u, v, w)$ of an ensemble of magnetic moments in a magnetic field $\vec{B} = (B_x, B_y, B_z)$ is determined by the Bloch equations,

$$\frac{d}{dt}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \gamma \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} \times \begin{pmatrix} u \\ v \\ w \end{pmatrix} - \begin{pmatrix} \gamma_2\, u \\ \gamma_2\, v \\ \gamma_1 (w - w_{\text{eq}}) \end{pmatrix},$$

where w_{eq} is the equilibrium z-component of $\vec{M}$, when all fields are 0; $\gamma_1$and γ_2 are called the longitudinal and transverse relaxation rates, respectively. The total magnetic field is the vector sum of a static field $\vec{B}_0$ along z and a field $\vec{B}_1$ rotating in the x-y plane,

$$\vec{B} = \vec{B}_0 + \vec{B}_1 = \begin{pmatrix} B_1 \cos(\omega t) \\ B_1 \sin(\omega t) \\ B_0 \end{pmatrix}.$$

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where $\omega_0 = \gamma |\vec{B}_0|$ is the free Larmor precession frequency around $\vec{B}_0$ and $\Omega_R = \gamma |\vec{B}_1|$ is called the Rabi frequency, which characterizes the magnitude of $\vec{B}_1$, that is, the strength of the interaction of the rotating field with the magnetization.

In the Demonstration, $\omega_0 = 1$, that is, all frequencies and relaxation rates are expressed in units of ω_0. The time unit is therefore $2\pi/\omega_0$ and the total time is equivalent to the number of Larmor cycles.

RELATED LINKS

- Bloch, Felix (1905-1983) (*ScienceWorld*)
- Nuclear Magnetic Resonance (*ScienceWorld*)

DOWNLOAD DEMONSTRATION SOURCE CODE »

PERMANENT CITATION DATA

"Magnetic Resonance and Bloch Equations" from The Wolfram Demonstrations Project
http://demonstrations.wolfram.com/MagneticResonanceAndBlochEquations/

- Contributed by: Gianni Di Domenico (Université de Neuchâtel) and Antoine Weis (Université de Fribourg)

E. M. Purcell

The Nobel Prize in Physics 1952

Biography

Edward Mills Purcell was born in Taylorville, Illinois, U.S.A., on August 30, 1912. His parents, Edward A. Purcell and Mary Elizabeth Mills, were both natives of Illinois. He was educated in the public schools in Taylorville and in Mattoon, Illinois, and in 1929 entered Purdue University in Indiana. He graduated from Purdue in electrical engineering in 1933.

His interest had already turned to physics, and through the kindness of Professor K. Lark-Horovitz he was enabled, while an undergraduate, to take part in experimental research in electron diffraction. As an Exchange Student of the Institute of International Education, he spent one year at the Technische Hochschule, Karlsruhe, Germany, where he studied under Professor W. Weizel. He returned to the United States in 1934 to enter Harvard University, where he received the Ph.D. degree in 1938. After serving two years as instructor in physics at Harvard, he joined the Radiation Laboratory, Massachusetts Institute of Technology, which was organized in 1940 for military research and development of microwave radar. He became Head of the Fundamental Developments Group in the Radiation Laboratory, which was concerned with the exploration of new frequency bands and the development of new microwave techniques. This experience turned out to be very valuable. Perhaps equally influential in his subsequent scientific work was the association at this time with a number of physicists, among them I.I. Rabi, with a continuing interest in the study of molecular and nuclear properties by radio methods.

The discovery of nuclear magnetic resonance absorption was made just after the end of the War, and at about that time Purcell returned to Harvard as Associate Professor of Physics. He became Professor of Physics in 1949; his present title is Gerhard Gade University Professor. He has continued to work in the field of nuclear magnetism, with particular interest in relaxation phenomena, related problems of molecular structure, measurement of atomic constants, and nuclear magnetic behaviour at low temperatures. He has made some contribution to the subject of radioastronomy.

He is a Fellow of the American Physical Society, a member of the National Academy of Sciences, of the American Academy of Arts and Sciences, and of the President's Science Advisory Committee under President Eisenhower from 1957-1960 and under President Kennedy as from 1960.

In 1937, Purcell married Beth C. Busser. They have two sons, Dennis and Frank.

EDWARD M. PURCELL

Research in nuclear magnetism

Nobel Lecture, December 11, 1952

Professor Bloch has told you how one can detect the precession of the magnetic nuclei in a drop of water. Commonplace as such experiments have become in our laboratories, I have not yet lost a feeling of wonder, and of delight, that this delicate motion should reside in all the ordinary things around us, revealing itself only to him who looks for it. I remember, in the winter of our first experiments, just seven years ago, looking on snow with new eyes. There the snow lay around my doorstep - great heaps of protons quietly precessing in the earth's magnetic field. To see the world for a moment as something rich and strange is the private reward of many a discovery. But I am afraid it has little bearing on the sober question we must, as physicists, ask ourselves: What can we learn from all this about the structure of matter? It is my privilege to tell you now of some of the things that can be learned.

Let us begin with the most direct application of nuclear induction methods, the measurement of nuclear magnetic moments. The basis for this is the resonance condition

$$f = \frac{\mu H_0}{Ih}$$

in which f is the frequency of precession of the axis of nuclear spin in a magnetic field of strength H_o, and μ is the magnetic moment of the nucleus. The number I is the nuclear spin quantum number, an integer or half-integer, and h is Planck's constant. Now H_o, except for a certain slight correction, is simply the field of the magnet in which the substance has been put, and it can be measured. The frequency of precession, once it is detected, is easily measured with high accuracy, and thus one can determine the quantity μ/Ih. However, for practical reasons, it is hard to measure the strength of a magnetic field very precisely. This difficulty is avoided if one is content to measure the *ratio* of the magnetic moments for two different nuclear species. We could, for example, compare the precession frequencies f_H and f_D for protons and deuterons exposed to the *same* magnetic field H_o. A mixture of light

Perhaps the greatest present need, through most of the Periodic Table, is for spin values and only moderately precise moment values, to help test the promising shell theories of nuclear structure. Nuclearinduction methods will surely continue to contribute to this task. Spin determinations in particular, which depend on careful intensity measurements, are likely to receive more attention.

The experimental physicist often needs to measure the intensity of a magnetic field. Until recently a precise magnetic field measurement has been a formidable undertaking, but it is such no longer. At the National Bureau of Standards in Washington, Hipple and his collaborators[3] have measured the proton precession frequency and, at the same time, have measured in absolute units the intensity of their magnetic field. All the resources of the Bureau were brought to bear on the latter measurement, and an accuracy of 1 part in 40,000 was achieved. Knowing their result, an experimenter anywhere in the world can determine absolute magnetic intensities to the same precision, using equipment no more elaborate than an ordinary radio receiver. He need only determine the proton precession frequency in the field in question. Few physical quantities are as easy to measure accurately as a radio frequency, thanks to modern electronic techniques and the availability, in most countries, of standard-frequency broadcasts.

Already a number of experiments have been performed in which the nuclear resonance has served as a standard of reference. Certain ones are of special interest because they have improved our knowledge of the fundamental atomic constants. In each of these experiments two *different* magnetic resonance phenomena have been observed at the same time in the same magnet. The phenomena involved are indicated in Fig. 2. The precession of the proton moment (Fig. 2 a) we have already described; it is detected and measured in a nuclear induction experiment. The electron has an intrinsic spin and magnetic moment and also precesses in a magnetic field (Fig. 2 b). For the same field strength, the electron's spin-precession frequency is about 700 times greater than the proton precession frequency. A bare proton, moving in a magnetic field, revolves in a circular orbit with a certain frequency (Fig. 2 c). This is the familiar principle of the cyclotron and we might name the associated frequency the « proton cyclotron frequency ». A free electron in a magnetic field likewise revolves in a circular orbit, with a frequency that we may call the « electron cyclotron frequency ». These « cyclotron frequencies » are governed simply by the particle's charge-to-mass ratio. If any pair of these resonance phenomena are compared, in the same magnetic field,

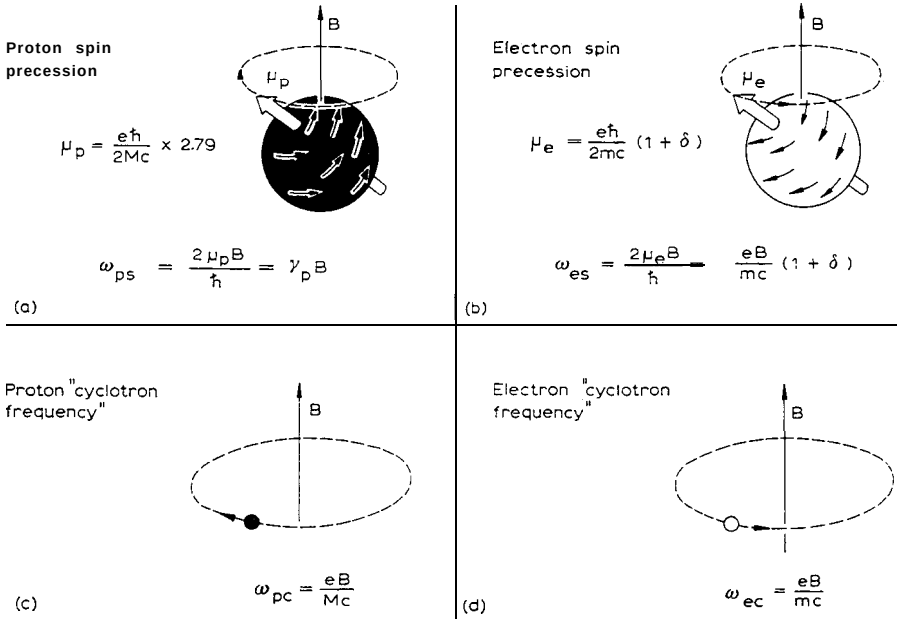

Fig. 2. Four elementary magnetic resonance phenomena.

the field strength cancels out and we are left with a relation between certain atomic constants. The experiment is thus reduced to the measurement of a frequency *ratio.*

The ratio of the proton spin-precession frequency to the proton cyclotron frequency has been determined by Hipple[4], and by Bloch[5], in ways very different but equally ingenious. (The direct result of this measurement is the value of the proton moment in nuclear magnetons.) In my laboratory we have measured the ratio of the proton precession frequency to the cyclotron frequency of free electrons[6]. The precession of the spin of a truly free electron has not been observed, but Kusch and his collaborators at Columbia, by combining the techniques of atomic beams and nuclear induction, have determined very precisely the ratio of the proton's precession frequency to that of the spinning electron in the hydrogen atom[7]. The results of these experiments have in one way or another improved the accuracy of the following constants of atomic physics: the Faraday, *F;* the specific charge of the electron, *e/mc;* the ratio of the mass of the proton to that of the electron, ***M/m;*** the dimensionless *fine-structure constant*, $2\pi e^2/hc$. They have also helped to test the new theoretical advances in quantum electrodynamics led by Schwinger.

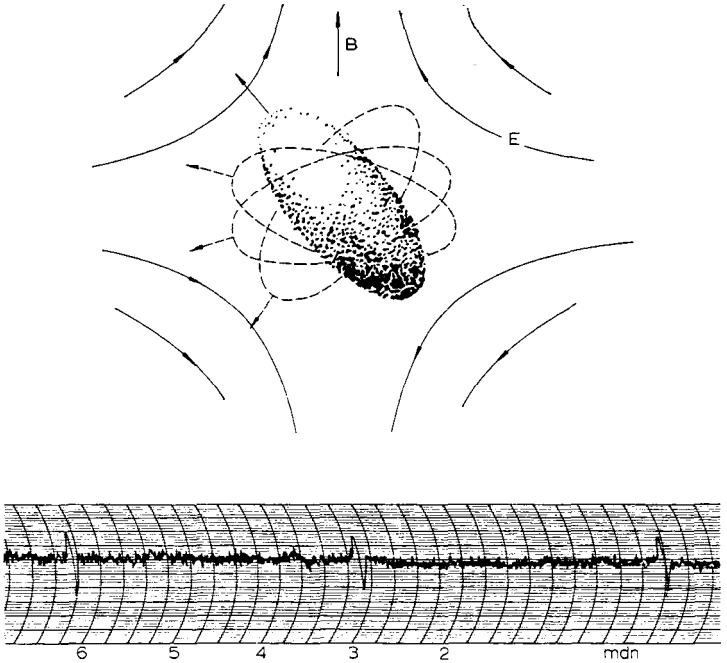

Fig. 4. A nucleus with an electric quadrupole moment in a uniform magnetic field (B) and a non-uniform electric field *(E). Lower trace:* ^{23}Na resonance in a single crystal of $NaNO_3$, showing splitting of magnetic resonance line into a triplet.

down to fractions of a millisecond in others. I show only one example, and that mainly to clarify the meaning of the term relaxation. Fig. 5 is a plot of the gradual approach to equilibrium magnetization of an originally unmagnetized crystal of ammonium bromide. This can be called, quite properly, a *cooling curve.* The nuclear spins, in order to align themselves with the magnetic field, must give off energy to the crystal lattice. They are coupled so weakly to the lattice that the transfer of energy takes more than several minutes. Actually the approach to equilibrium is exponential, and the corresponding time constant, in this example *32* seconds, is what one calls the spin-lattice relaxation time.

It may seem astonishing, at first, that so long a time is associated with an atomic process. But in fact, if one looks at the problem more closely, it is hard to understand why the time is not much *longer.* Moreover this crystal, at a somewhat higher temperature, has a relaxation time of only 0.01 seconds! The explanation of this, and of similar behavior in a wide class of substances, lies in internal molecular motions other than mere vibration. It is

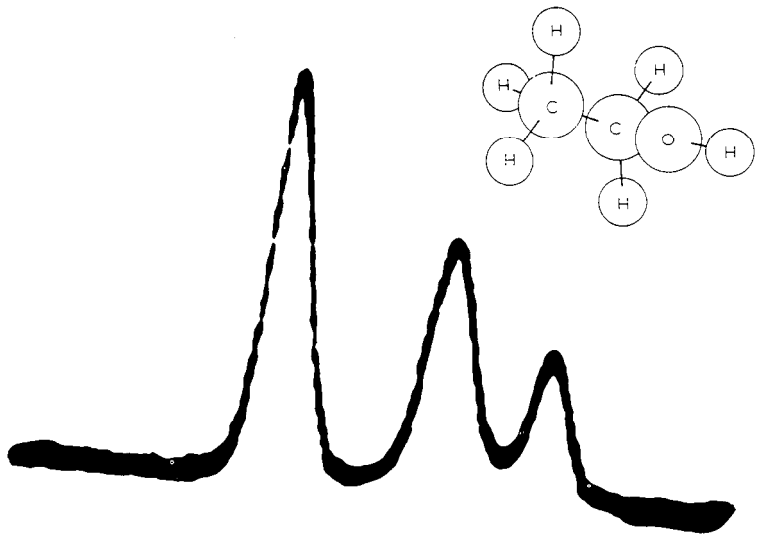

Fig. 6. The proton resonance in ethyl alcohol, observed with high resolution. The three lines arise from the CH_3 hydrogens, from the CH_2 hydrogens, and from the OH hydrogen, respectively.

There remain several puzzling aspects of nuclear relaxation. In trying to understand them we find ourselves still returning to some of the ideas advanced twenty years ago by Professor Waller. While I speak of theoretical contributions to nuclear magnetism, I must mention also Professor Van Vleck, who has put the theory of line-width on a rigorous basis, a notable advance.[15]

It is an old story in physics that higher resolving power leads to new effects. We remember that the magnetic moment of the nucleus was itself discovered through the hyperfine structure of lines in the visible spectrum. The nuclear resonance line in a liquid or gas can be remarkably narrow, as you have already seen. As soon as the reason for this was recognized, it became clear that the only practical limit on resolution was the inhomogeneity of the magnetic field applied to the specimen. Efforts were made in many laboratories to improve the magnets, and to use smaller specimens as well. With the improved resolution, it was found that identical nuclei, in the same applied field but in chemically different molecules, do not precess at exactly the same frequency. The explanation is simple: the magnetic field at an atomic nucleus differs slightly from the field externally applied because of the shielding effect of the electron cloud around the nucleus. In different molecules the atom's electron configuration will differ slightly, reflecting differences in

Felix Bloch

The Nobel Prize in Physics 1952

Biography

Felix Bloch was born in Zurich, Switzerland, on October 23, 1905, as the son of Gustav Bloch and Agnes Bloch (*née* Mayer). From 1912 to 1918 he attended the public primary school and subsequently the "Gymnasium" of the Canton of Zurich, which he left in the fall of 1924 after having passed the "Matura", i.e. the final examination which entitled him to attend an institution of higher learning.

Planning originally to become an engineer, he entered directly the Federal Institute of Technology (Eidgenössische Technische Hochschule) in Zurich. After one year's study of engineering he decided instead to study physics, and changed therefore over to the Division of Mathematics and Physics at the same institution. During the following two years he attended, among others, courses given by Debye, Scherrer, Weyl, as well as Schrödinger, who taught at the same time at the University of Zurich and through whom he became acquainted, toward the end of this period, with the new wave mechanics. Bloch's interests had by that time turned toward theoretical physics. After Schrödinger left Zurich in the fall of 1927 he continued his studies with Heisenberg at the University of Leipzig, where he received his degree of Doctor of Philosophy in the summer of 1928 with a dissertation dealing with the quantum mechanics of electrons in crystals and developing the theory of metallic conduction. Various assistantships and fellowships, held in the following years, gave him the opportunity to work with Pauli, Kramers, Heisenberg, Bohr, and Fermi, and to further theoretical studies of the solid state as well as of the stopping power of charged particles.

Upon Hitler's ascent to power, Bloch left Germany in the spring of 1933, and a year later he accepted a position which was offered to him at Stanford University. The new environment in which he found himself in the United States helped toward the maturing of the wish he had had for some time to undertake also experimental research. Working with a very simple neutron source, it occurred to him that a direct proof for the magnetic moment of the free neutrons could be obtained through the observation of scattering in iron. In 1936, he published a paper in which the details of the phenomenon were worked out and in which it was pointed out that it would lead to the production and observation of polarized neutron beams. The further development of these ideas led him in 1939 to an experiment, carried out in collaboration with L.W. Alvarez at the Berkeley cyclotron, in which the magnetic moment of the neutron was determined with an accuracy of about one percent.

During the war years Dr. Bloch was also engaged in the early stages of the work on atomic energy at Stanford University and Los Alamos and later in counter-measures against radar at Harvard University. Through this latter work he became acquainted with the modern developments of electronics which, toward the end of the war, suggested to him, in conjunction with his earlier work on the magnetic moment of the neutron, a new approach toward the investigation of nuclear moments.

These investigations were begun immediately after his return to Stanford in the fall of 1945 and resulted shortly afterward in collaboration with W.W. Hansen and M.E. Packard in the new method of nuclear induction, a purely electromagnetic procedure for the study of nuclear moments in solids, liquids, or gases. A few weeks after the first successful experiments he received the news of the same discovery having been made independently and simultaneously by E.M. Purcell and his collaborators at Harvard.

FELIX BLOCH

The principle of nuclear induction

Nobel Lecture, December 11, 1952

It is a tribute to the inherent harmony and the organic growth of our branch of science that every advance in physics is largely due to the developments that preceded it. The discovery for which Purcell and I have received the honor of the Nobel Prize award for the year 1952 is a typical example of this situation, and before describing the principle I shall therefore present an outline of its long and distinguished background.

Both the method and the object go back ultimately to spectroscopy, a field to which modern physics owes so much in other respects. Among the various aspects of this field there are two which are of particular importance here: the Zeeman effect for introducing magnetic fields as an essential element of spectroscopy, and the hyperfine structure of spectral lines for revealing the existence of nuclear moments. The correct interpretation of hyperfine structures was first given in 1924 by Pauli[1], who proposed that atomic nuclei may possess an intrinsic angular momentum (spin) and, parallel to its orientation, a magnetic moment. The energy of interaction of this magnetic moment with the magnetic field $H_{(0)}$, produced by the atomic electrons at the position of the nucleus, depends upon the angle between them and leads thus to the observed small splitting of the energy levels. Conversely, it is possible under suitable conditions to determine from this splitting both the spin and the magnetic moment of the nucleus, and these two important quantities have indeed been determined in a great number of cases from the observation of hyperfine structures. The magnetic moments of the nuclei have been found, in all observed cases, to be of the order of the « nuclear magneton » which one obtains by substituting in the formula for the atomic Bohr magneton the mass of the proton in place of that of the electron. Nuclear moments are thus about a thousand times smaller than atomic moments, and this is plausible in view of the fact that one deals here with protons instead of electrons as elementary charged constituents. There are, however, distinct disadvantages in the optical determination of nuclear moments. In the first place the accuracy is seriously limited due to the fact that the effect consists only in such a small splitting of spectral lines that one has to

through a region where a weak oscillating field is superimposed on a strong constant field, provided that the frequency of the former is equal to the frequency with which the neutron moment carries out a precessional motion around the direction of the constant field. A knowledge of this field and of the corresponding resonance frequency directly determines the magnetic moment under the safe assumption that the spin of the neutron is 1/2, and the magnetic scattering effect enters in this arrangement merely as an indicator for the occurrence of resonance depolarization. The application to polarized neutron beams was also noted by Rabi[7] in his previously mentioned original paper on the magnetic resonance method. It was first achieved in 1939 by Alvarez and myself[11] with the use of the Berkeley cyclotron, and yielded a value for the magnetic moment of the neutron which was consistent with that of the deuteron if one assumed the latter to be additively composed of the moments of the proton and the neutron. The accuracy of this measurement amounted to about one percent and was partly limited by that with which the strength of the constant field could be determined. Another limit of accuracy arose from the smallness of the observed polarization effect, but a subsequent systematic investigation of neutron polarization[12], carried out with the Stanford cyclotron, showed how this effect could be greatly increased.

It was of considerable importance to improve the accuracy of the determination of the neutron moment to at least one part in a thousand in order to test small deviations from the additivity of the moments of the proton and the neutron, which could be expected in connection with the finite electric quadrupole moment of the deuteron, according to the theoretical work of Rarita and Schwinger[13]. The fact that higher accuracy hinged essentially upon that of a field calibration and the search for a suitable and convenient standard led me to new ideas when, toward the end of the last War, my thoughts turned back to the continuation of my previous work.

The essential fact of the magnetic resonance consists in the change of orientation of nuclear moments, and the methods to be employed in molecular and atomic beams as well as in neutron beams are primarily indicated by the different ways to detect this change. The acquaintance with radio techniques during the War suggested to me still another and much simpler way, that of detecting the reorientation of nuclear moments through the normal methods of radio reception. The signals to be detected would be due to the electromagnetic induction caused by nuclear reorientation and should appear as a voltage difference between the terminals of an external electric circuit.

macroscopic point of view, one thus deals with a situation in which the protons in our cubic centimeter of water have the effect of an invisible compass needle rotating in its interior. The « invisibility » refers actually only to observation of optical frequencies; the rotation occurs in the range of radio-frequencies, and it can very well be observed by using Faraday's law of induction. Indeed, the rotation of our compass needle is accompanied by that of a magnetic field which possesses an alternating component perpendicular to the axis of rotation, and hence by an electromotive force, induced in a suitably wound coil of wire around the sample. From here on it is merely a matter of the standard techniques of radio reception to rectify and amplify this electromotive force so that it can be recorded on a volt-meter, displayed on a cathode-ray oscillograph, or made audible in a loudspeaker.

What amazed me most in my first calculations on this effect was the magnitude of the signals which one could expect from nuclear induction. In our example of a cubic centimeter of water in a normal field of a few thousand gauss they turned out to amount to the order of a millivolt. This magnitude is well above the noise which accompanies any radio receiver and which sets the ultimate limit of signal detection. It should be observed here that, being a phenomenon of fluctuations, the noise can always be reduced by averaging over sufficiently long times. This procedure was used later to very greatly increase the sensitivity of the method; it is characteristic of the present possibilities that my collaborators have succeeded in the last few years in detecting in natural water signals arising from deuterium and from the isotope of oxygen with atomic mass 17, despite their low abundances of 0.02 and 0.04 percent, respectively.

The existence and detection of a precessing nuclear polarization in a sample represents to my mind the basis of nuclear induction. It is, however, necessary to consider also the features which produce and counteract the tilt of the polarization with respect to the constant field. Magnetic resonance enters here as the most important means of producing the tilt, since it allows its achievement under the application of relatively weak oscillating fields. In fact, it is a common feature of every resonance phenomenon that relatively weak external forces can produce large effects if their frequency is equal to the natural frequency of the system to which they are applied. The natural frequency in question is, in our case, that with which the nuclear polarization precesses by itself around the constant field and the practical way to determine this frequency is to vary either that of the applied alternating field or the magnitude of the constant field until resonance conditions are

established and detected by a maximum of the observed nuclear induction signal. The simultaneous knowledge of resonance field and frequency then directly yields, as in the use of magnetic resonance in molecular beams, the gyromagnetic ratio and, with a knowledge of the spin, the magnetic moment of the nucleus. Actually, it is also possible to determine the spin separately by using the additional piece of information contained in the intensity of the observed signal.

To follow the analogue of mechanical resonance we must now come back to relaxation, which can be seen to act like a friction, and which counteracts the tilt produced by the alternating field. If the friction is large, i.e., if the relaxation time is short, it will either reduce the effect for a given amplitude or require a correspondingly larger amplitude of the alternating field. It will, in either case, result in a relatively broad resonance line, thus diminishing the accuracy of the measurement. While from this point of view it is undesirable to have too short a relaxation time, it is equally undesirable to have it too long, since the very circumstance of producing its tilt diminishes the magnitude of the polarization so that it requires the refreshing influence of the relaxation mechanism to bring it back to its equilibrium value.

There was not much known about the magnitude of nuclear relaxation times when Purcell and I started our first experiments on nuclear induction, and the main doubt about their success arose from the possibility of insufficient relaxation. In fact, it seems, in retrospect, that the failure of Gorter's first attempt[6], as well as of a second one, undertaken in 1942[14], was primarily due to this circumstance. While E. M. Purcell, H. C. Torrey and R. V. Pound[15], toward the end of 1945, obtained their first positive results from protons in paraffin, the late W. W. Hansen, M. E. Packard, and I[16] found ours a few weeks later in water without either of the groups knowing anything about the work of the other. The relaxation time of paraffin has the convenient value of about $\frac{1}{100}$ second, while pure water has a somewhat unfavorably long relaxation time of about 2 seconds. Neither of these two values had been foreseen, and I was fully prepared to find the relaxation time of pure water considerably longer and in fact too long for our method of observation. It was known, however, that the conversion of ortho- and para-hydrogen was accelerated by the presence of paramagnetic atoms and molecules; this mechanism has the common feature, with the attainment of the equilibrium polarization of protons, that it requires a random process of nuclear reorientation, and it had been understood to take place through the magnetic field of the paramagnetic catalyst acting upon the magnetic mo-

The description of nuclear induction which I have presented follows closely my own original thoughts on the subject but it can equally well be approached from other angles. The simplest one is probably that of Gorter[6] in his first attempt to detect nuclear magnetic resonance. We have seen before that the alternating field tilts the nuclear polarization against the constant field. This process requires a certain amount of work which, through relaxation, will reappear in the form of heat produced in the sample. The effect in fact does not involve induction but represents pure nuclear resonance absorption; however, it would be very slight and has not yet been established. A second attempt of Gorter[14], carried out later, is based upon the fact that the nuclear paramagnetic susceptibility has a maximum for radiofrequencies, corresponding to magnetic resonance conditions; it would manifest itself in the frequency of an electric oscillator of which a coil, surrounding the sample, forms the self-inductance. This scheme is actually one of the many others which can be devised for the observation of nuclear induction and, if successful, would have represented the first demonstration of the effect. Purcell's first successful experiment involved the electrodynamical aspect of absorption insofar as its occurrence under resonance conditions was manifested through the increased loss of a cavity resonator; the cavity was replaced in his succeeding arrangements by more conventional circuit elements. A particularly suitable and convenient arrangement consists of a radiofrequency bridge, which contains in one arm a coil, surrounding the sample. As a consequence of nuclear induction there occurs, under resonance conditions, a change of the impedance of this coil and thereby a readily detectable change in the balance of the bridge. It should be remarked that the change of impedance is complex, with its real part corresponding to absorption, its imaginary part to dispersion. This fact can be traced back to the phase relation between the nuclear induction signal and the applied radiofrequency field, and the phase sensitivity of the bridge allows the observation of the effect either as absorption or as dispersion or as a combination of both.

Finally, I shall give a brief description of our own original arrangement which we still use in its principal features. The essential balance which Purcell has obtained by a bridge method is here to a large extent achieved geometrically by using two radiofrequency coils with their axes oriented at right angles to each other and to the constant field. One of them, the « transmitter coil » produces the alternating field, while the other, the « receiver coil », serves for detection of the nuclear induction signal (see Figs. 1 and 2). A small amount of coupling between the two coils is admitted to produce

a voltage across the receiver coil, and its phase with respect to the superimposed voltage induced by the nuclei can be adjusted for the observation of either absorption or dispersion in similarity to the bridge method (see Figs. 3 and 4).

A considerable variety of other circuits has been introduced by different investigators. Except for the greater or lesser ease of avoiding instrumental difficulties, they lead to the same ultimate sensitivity and accuracy of the method, since they all observe the same basic phenomenon.

There is, however, one distinctive feature in the crossed-coil arrangement, which automatically yields another significant piece of information. The two coils imply a sense of rotation around the constant field; depending upon whether the nuclear polarization precesses in the same or the opposite sense of rotation there results a phase difference of 180 degrees between the voltage in the receiver coil due to coupling with the transmitter coil and the superimposed voltage due to nuclear induction. The action of the rectifier translates this phase difference into an inversion of the signal, which is directly displayed on the oscillograph or on the recording instrument. One obtains in this simple manner information about the sign of the magnetic moment of the nucleus, defined by its relative orientation to the angular momentum, since it is this sign which determines the sense of rotation of the nuclear polarization in a given field. The sign of nuclear moments represents an important clue to their interpretation in terms of nuclear structures; usually it is referred to the sign of the proton moment, which has been known for a considerable time to be positive. It has been determined in this manner for a number of nuclei where it was not previously known.

1. W. Pauli, *Naturwiss.*, 12 (1924) 741.
2. R. Frisch and O. Stern, Z. *Physik*, 85 (1933) 4.
   I. Estermann and O. Stern, Z. *Physik*, 85 (1933) 17.
   I. Estermann and O. Stern, *Phys. Rev.*, 46 (1934) 665.
3. G. Breit and I. I. Rabi, *Phys. Rev.*, 38 (1931) 2082.
4. E. Fermi, *Z. Physik*, 60 (1930) 320.
5. J. M. B. Kellogg, I. I. Rabi, N. F. Ramsey, and J. R. Zacharias, *Phys. Rev.*, 55 (1939) 318; 57 (1940) 677.
6. C. J. Gorter, *Physica*, 3 (1936) 995.
7. I. I. Rabi, *Phys. Rev.*, 51 (1937) 652.
8. P. A. M. Dirac, *Proc. Roy. Soc. London*, 117 (1928) 610.

Otto Stern

The Nobel Prize in Physics 1943

Biography

Otto Stern was born in Sorau, Upper Silesia, Germany, on February 17, 1888. In 1892 he moved with his parents to Breslau, where he attended high school. He began to study physical chemistry in 1906, receiving his Ph.D. degree from the University of Breslau in 1912. In the same year he joined Einstein at the University of Prague and later followed him to the University of Zurich, where he became Privatdocent of Physical Chemistry at the Eidgenössische Technische Hochschule in 1913.

In 1914 he went to the University of Frankfurt am Main as Privatdocent of Theoretical Physics, remaining there until 1921, except for a period of military service. From 1921 to 1922 he was Associate Professor of Theoretical Physics at the University of Rostock, becoming, in 1923, Professor of Physical Chemistry and Director of the laboratory at the University of Hamburg, where he remained until 1933. In that year he moved to the United States, being appointed Research Professor of Physics at the Carnegie Institute of Technology, Pittsburgh where he remained until 1945, then becoming professor emeritus.

His earliest work was in the field of theoretical physics, especially that of statistical thermodynamics and quantum theory, on which he has published important papers. After 1919, his attention was directed more to experimental physics. His development and application of the molecular beam method proved to be a powerful tool for investigating the properties of molecules, atoms and atomic nuclei. One of the early applications of this was the experimental verification of Maxwell's law of velocity distribution in gases. He collaborated with Gerlach to work on the deflection of atoms by the action of magnetic fields on their magnetic moment, then went on to measure the magnetic moments of sub-atomic particles, including the proton. His work on the production of interference by rays of hydrogen and helium was a striking demonstration of the wave nature of atoms and molecules.

Dr. Stern was awarded an LL.D. by the University of California, Berkeley, 1930. He is a member of the National Academy of Sciences (USA), the American Association for the Advancement of Science, and the Philosophical Society. He holds foreign membership of the Royal Danish Academy of Sciences. He lives at Berkeley, California.